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Damping Augmentation for Three-Cable Suspension System for Structural Testing

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An approach to the simultaneous design of multiple dampers for damping augmentation of a three-cable suspension system for validation testing of space structures is presented. The system features a three-cable platform with three onboard inertial dampers directed along the x, y, and z axes. These triaxial dampers have been used in suppressing pendulum vibrations of the suspension system. The primary objective is to optimally design three dampers in a simultaneous procedure, in which a multimode tuning method is developed to minimize a cost function of vibration effects simulated from a multibody dynamics model. This design method is formulated as a minimum-time control problem and solved using an efficient optimization procedure. It is shown that such a multimode tuning technique leads to simultaneous tuning of three inertial dampers with better optimality than for a conventional, single-mode tuning method. The simulated time histories for vibration suppression in a three-cable suspension system are also presented.

Nomenclature

A_c = null-space matrix of Jacobian matrix B_c
A_o = null-space matrix of Jacobian matrix B_o
B = constraint Jacobian matrix
B_o, B_c, B_n = submatrices of B
$\bar{C}$ = lumped matrix of $A_c^T A_o^T M(\dot{A}_o A_c + A_o \dot{A}_c)$
c_i = damping coefficients of inertial dampers
F = forcing vector of equations of motion, $[f_1, \ldots, f_7]^T = [f_{ix}, f_{iy}, f_{iz} - m_i g, \tau_i - \tilde{\omega} j_i \omega]^T$
$F(y)$ = cost function of design variables
$\mathcal{F}_i$ = natural frequencies of vibrational modes
$\mathcal{F}_{pend}$ = natural frequency of a simple pendulum, $(1/2\pi)\sqrt{(g/h)}$
f_i = subvectors of F
f_{ix}, f_{iy}, f_{iz} = interactive forces between springs and dashpots
h = length of a simple pendulum
I_3 = moment of inertia of damper 3
j_i = moment-of-inertia matrix of the ith body
k_i = spring constants of inertial dampers
l_i = lengths of cables
$\tilde{l}$ = skew-symmetric matrices of position vectors
M = mass matrix of equations of motion
$\bar{M}$ = transformed mass matrix, $A_c^T A_o^T M A_o A_c$
m_i = mass matrix of the ith body
R_i = rotational matrix of the ith body
r_i = position vector of the ith body
t = time
$\bar{U}$ = product of null-space matrices, $A_c^T A_o^T$
x_i, y_i = body coordinates of the ith body
y = vector of design variables, $[k_1, c_1, k_2, c_2, k_3, c_3]^T$
y^0 = vector of initial values of y
y_{opt} = vector of optimum solution of design variables
ε = distance between c.g. and point P in Fig. 3
λ, λ_c = Lagrange multipliers
ξ = vector of state variables, $[\dot{r}_1, \omega_1, \ldots, \dot{r}_7, \omega_7]^T$
ξ_ω, ξ_i = vectors of state variables after transformation
ρ = gyration radius, $\sqrt{(I/m)}$
Φ = nonholonomic constraint matrix
$\phi(y)$ = constraint function in optimization program
ϕ, ϕ_1 = angles of cables
ψ_1 = angle of cable
ω_i = components of angular velocity

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Introduction

DAMPING augmentation for cable suspension systems, which usually experience pendulum vibrations during the ground testing of flexible space structures, has received increasing attention in recent years.[1–4] Studies performed so far indicate that the pendulum vibrations triggered during suspending can pose a severe restriction on the precision assessment of the structural characteristics. In response, several inertial dampers[1–7] have been used, either passively or actively, as a way to augment the pendulum damping in cable suspension systems for vibration suppression purposes. Active inertial dampers, in general, appear superior to their passive counterparts in robustness; however, some mechanical systems are difficult to control by such means because of complexity in modeling them. In view of this fact, it is desirable to enhance system damping through a passive approach to improve the dynamic response prior to finalizing an active control design.

Inertial dampers have been very effectively used for damping augmentation in small-scale systems, mostly ones subjected to single-axis motion.[2,5–7] For instance, a single ball-in-tube nutation damper[5] has been installed on a navigation gyroscope to enhance gyroscopic damping against wobbly motion in a single direction. Recently, NASA has adopted nine collocated accelerometer–thruster pairs as virtual vibration absorbers for a computation–structure–integration (CSI) evolutionary model.[1] In this CSI testbed conducted at NASA, each absorber pair is separately designed so that its frequency is tuned to that of a single mode as targeted, following damping optimization through the use of a root-locus plot. Such a single-mode tuning method is also very often called for in other damper designs.[5–7] Although this method does provide positive damping optimality for the system, its use has largely been limited to single-axis damper design targeting a single mode at a time. This method, therefore, is not entirely satisfactory for systems that require simultaneous design of multiaxial dampers, as is necessary for dealing with multiple modes interacting with each other.

The objective of this paper is thus to introduce an optimal design technique for damping augmentation and demonstrate the concept as applied to a three-cable suspension system with three onboard inertial dampers subjected to the triaxial motion. Figure 1 shows a perspective of this damper-equipped system that can be applied as a means of suspension for space structures during preflight testing.[2–4] Many questions arise out of such a generic system under the proposed damper design. First, it needs to be determined if the multiaxial dampers are feasible for damping augmentation

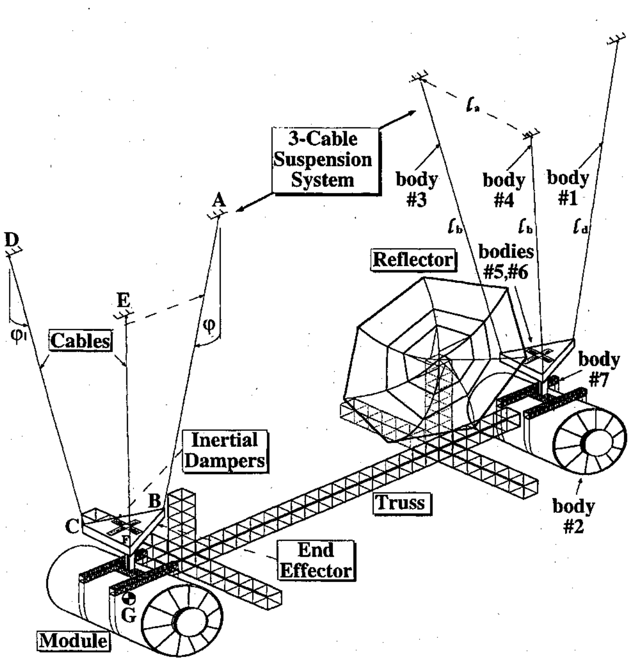


Fig. 1 Three-cable suspension system with inertial dampers.

of a three-cable suspension system. Then, there is the question of whether it is indeed possible to bring about the simultaneous design of three independent dampers in an integrated manner. Above all, the performance of the newly modified damper-equipped system needs to be shown to be superior to that of a system based on the classical approach.

The outline of the remainder of this paper follows. First, a description of the three-cable suspension system is presented. Second, the derivation of the equations of motion of this system is detailed. Then, application of a single-mode tuning method for damper design is stated, followed by the introduction of a multimode tuning method proposed for designing three inertial dampers in a simultaneous manner. Finally, simulation results on damped and undamped vibration are presented for discussion.

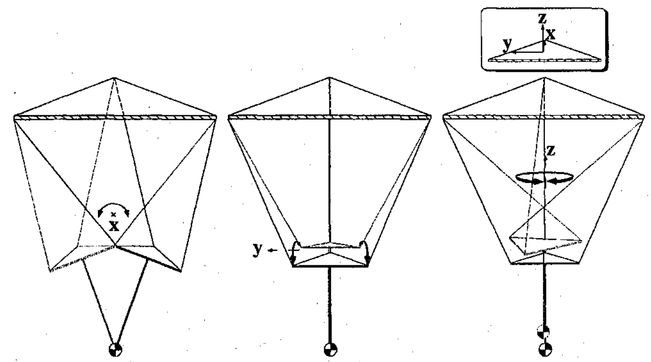
Description of System

The problem of stabilizing the cable suspension system by means of damping augmentation has inspired the introduction of a simple device made of multiaxial inertial dampers for eliminating the attendant pendulum vibrations. An investigation into its applicability and its simultaneous design is the subject of this paper. We shall begin with a brief description of the cable suspension system in conjunction with the multiaxial inertial dampers, to serve as background.

Three-Cable Suspension System

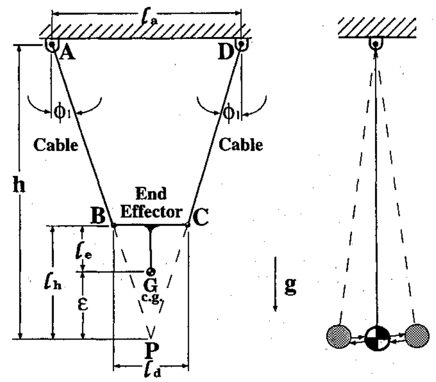
The cable suspension system is made of a set of tension members configured to hold the payload with the lowest possible load in each member, in the presence of a gravitational field. It is thought to be the most efficient structural form for carrying large steady loads, in view of the light weight and high tensile strength of the cable. Several cable suspension systems, in use on Earth or in space, have been studied by Mikulas and Yang.[3] Among them, an alternative suspension concept with three cables is regarded as the most efficient in the sense of suspending the payload with the minimal number of cables. Figure 1 shows a three-dimensional perspective of such a three-cable suspension system in static equilibrium. The cylindrical module is statically equilibrated by a triangular framework formed from the three connecting cables, and its center of gravity (c.g.) is located where the extension lines of three cables intersect. In this way, one obtains four concurrent forces in static equilibrium such that the gravitational force acting upon the module is counterbalanced.

The static equilibrium of this three-cable suspension device will not persist if perturbed by any external disturbances. Applying a Euler–Savary equation,[2,8] it has been found that if the module is



a) Yaw mode b) Pitch mode c) Roll mode

Fig. 2 Three pendulum modes of three-cable suspension system.



Cable suspension model Swinging pendulum

Fig. 3 Planar model of three-cable suspension system.

located with its c.g. inside an inflection sphere centered at the common point of the three cables, then the suspension system will experience a stable vibration when displaced, like that of a pendulum swinging in space. Since the system has three degrees of freedom geometrically constrained, the remaining three introduce three kinds of pendulum modes (yaw, pitch, roll), as illustrated in Figs. 2a, 2b, and 2c, respectively. Based on the model shown in Fig. 3, their natural frequencies have been analyzed[2,8] and expressed by the following closed-form equations.

Pitch and yaw modes:

$$\mathcal{F}_1 = \sqrt{\frac{[l_g/(h-l_g)]\left[l_g h + \left(l_a^2/4\right)\right] + l_e h}{\varepsilon^2 + \rho^2}}\,\mathcal{F}_{\text{pend}} \tag{1}$$

Roll mode:

$$\mathcal{F}_2 = \sqrt{\frac{(l_d/2)^2 + (l_d/2)h\tan\psi_1}{\rho^2}}\,\mathcal{F}_{\text{pend}} \tag{2}$$

As can be seen in Fig. 2, the three-cable suspension system displays three principal pendulum modes of vibration. Therefore, in considering passive vibration suppression for the system, one needs at least three distinct dampers set parallel to the axes of the pitch, roll, and yaw modes, respectively. For this purpose, a multiaxial damping mechanism is desirable and will be discussed.

Multiaxial Inertial Dampers

The inertial damper is an inertial-force-producing device that can be viewed as an energy sink in the system to dissipate the mechanical energy by moving (translating or rotating) the proof mass in a certain manner within a constrained domain. Several inertial devices have been used as an effective means to augment the system damping for vibration suppression purposes. However, many of the existing papers[1,5–7] on inertial dampers have emphasized classical single-degree-of-freedom vibration theory. A simultaneous approach, which incorporates the optimal design of multiaxial

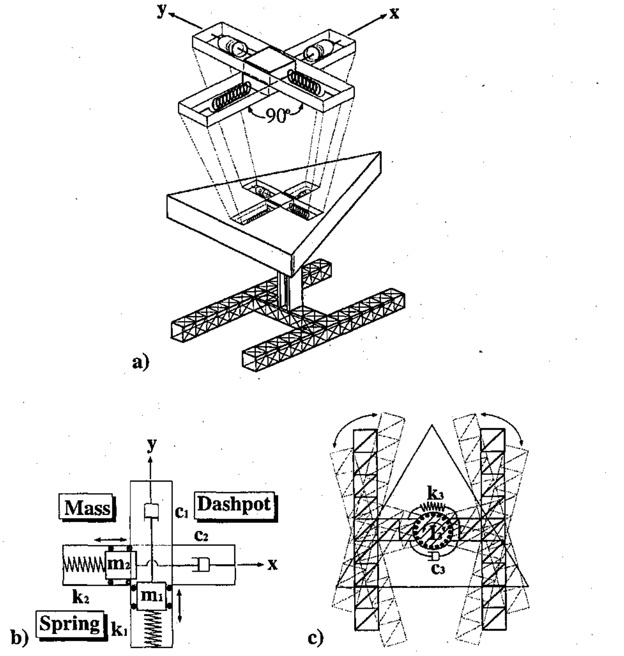


Fig. 4 Multiaxial inertial damper device.

inertial dampers and the multiple-degree-of-freedom systems, has not been attempted. Moreover, the incorporation of such a multiaxial damper device permits intrastructural redesign for damping enhancement with a view to improved dynamic response of complex systems, and it is this objective that forms the basis of this investigation.

Figure 4 shows the configuration of three inertial dampers mounted near the centroid of an end effector along the three principal axes. A spring–mass–dashpot assembly is used as the inertial damper because of its simplicity. In Fig. 4, dampers 1 and 2 are of translational type for the yaw and pitch modes, respectively, and damper 3 is of rotational type for the roll mode. Also, observe that the platform plays a dual role of 1) the end effector accommodating three inertial dampers and 2) the proof mass for damper 3. The three inertial dampers may be thought of as decoupled in the sense that each one provides an independent energy sink; however, they are virtually linked from the viewpoint of multibody dynamics. Hence, this triaxial damper device, if properly designed, is capable of augmenting damping for the three-cable suspension system against the attendant vibrations of the three pendulum modes as excited.

System Dynamics

In this section, the dynamics of a three-cable suspension system is studied by incorporating three inertial dampers into a tripod configuration, as shown in Fig. 1. Assume that bodies 1, 3, 4 represent the three cables $\overline{AB}$, $\overline{CD}$, $\overline{EF}$; body 2 represents the end effector $\triangle BCF$; and bodies 5, 6, 7 represent three inertial dampers along the x, y, z axes, respectively. The joints between bodies, at locations A to F, are spherical joints, whereas those associated with inertial dampers are treated as cylindrical joints. Applying d'Alembert's principle of virtual work,[9,10] the equations of motion of the three-cable suspension system in conjunction with three inertial dampers can be derived in the following matrix form:

$$M\ddot{\xi} + B^T\lambda = F \qquad \text{and} \qquad \Phi = B\dot{\xi} = 0 \tag{3}$$

where the mass matrix M can be written as

$$M = \mathrm{Diag}[m_1, j_1, \ldots, m_7, j_7] \tag{4}$$

where j_i are matrices containing the moments of inertia of the ith body, and m_i are component mass matrices. We denote by f_{ix}, f_{iy}, and f_{iz} the components of the interactive forces, which can be expressed by

$$\begin{aligned}
f_{5x} &= k_1\,\Delta x_{25} + c_1\,\Delta\dot{x}_{25}, & f_{6x} &= k_2\,\Delta x_{26} + c_2\,\Delta\dot{x}_{26}\\
f_{5y} &= k_1\,\Delta y_{25} + c_1\,\Delta\dot{y}_{25}, & f_{6y} &= k_2\,\Delta y_{26} + c_2\,\Delta\dot{y}_{26}\\
f_{5z} &= k_1\,\Delta z_{25} + c_1\,\Delta\dot{z}_{25}, & f_{6z} &= k_2\,\Delta z_{26} + c_2\,\Delta\dot{z}_{26}\\
&\tau_{7z} = k_3\,\Delta\theta_{27z} + c_3\,\Delta\omega_{27z} & &\\
f_{2x} &= -f_{5x} - f_{6x}, & f_{2y} &= -f_{5y} - f_{6y}\\
f_{2z} &= -f_{5z} - f_{6z}, & \tau_{2z} &= -\tau_{7z}
\end{aligned} \tag{5}$$

where the relative displacements are $\Delta x_{25} = x_2 - x_5$, $\Delta y_{25} = y_2 - y_5$, $\Delta z_{25} = z_2 - z_5$, $\Delta x_{26} = x_2 - x_6$, $\Delta y_{26} = y_2 - y_6$, $\Delta z_{26} = z_2 - z_6$, $\Delta\theta_{27z} = \theta_{2z} - \theta_{7z}$.

The Jacobian matrix B was also derived and is given in the appendix. Note that the Jacobian matrix as given by Eq. (A.1) consists of two submatrices B_o and B_c. Both B_o and B_c can be explicitly nullified from Eq. (3) by using a null-space method along with a natural partitioning scheme.[10] In doing so, a complement of B_o is first determined in the following variable transformation:

$$\dot{\xi} = A_o\dot{\xi}_\omega \qquad \text{and} \qquad \dot{\xi}_\omega = [\omega_1, \omega_2, \omega_3, \omega_4, \dot{x}_5, \omega_5, \dot{y}_6, \omega_6, \omega_7]^T \tag{6}$$

where the transformation matrix A_o is a null-space matrix, so that the orthogonality relation $B_oA_o = 0$ holds. Premultiplying Eq. (3) with the transpose of A_o and substituting for $\ddot{\xi}$ with $\dot{\xi}_\omega$ and $\ddot{\xi}_\omega$ derived from Eq. (6), one can obtain

$$A_o^TMA_o\ddot{\xi}_\omega + A_o^TM\dot{A}_o\dot{\xi}_\omega + B_n^T\lambda_c = A_o^TF \tag{7}$$

where B_n denotes the remaining Jacobian matrix as shown in the appendix. Similarly, one can explicitly determine another null-space matrix A_c according to the orthogonality $B_nA_c = 0$, and the resulting matrix makes the following transformation feasible:

$$\dot{\xi}_\omega = A_c\dot{\xi}_i$$
$$\text{and} \qquad \dot{\xi}_i = [\omega_{1z}, \omega_{2x}, \omega_{2y}, \omega_{2z}, \omega_{3z}, \omega_{4z}, \dot{x}_5, \dot{y}_6, \omega_{7z}]^T \tag{8}$$

which results in a set of nine independent variables including the variables of three inertial dampers, i.e., x_5, y_6, and ω_{7z}. Equation (7) is then transformed into

$$\bar{M}\ddot{\xi}_i + \bar{C}\dot{\xi}_i = \bar{U}F \tag{9}$$

Equation (9) thus provides a set of λ-free differential equations capable of implementing the simulations of a three-cable suspension system equipped with dampers, subjected to any external excitations.

Optimal Design of Dampers

This section presents two different methods for determining the parameters of three inertial dampers for the three-cable suspension system, including a single-mode and a multimode tuning method. The single-mode tuning method has been shown to be very useful for the design of a one-degree-of-freedom vibration absorber provided that the vibrational modes of the system are accessible. On the other hand, the multimode tuning method is made capable, by an optimizer, of dealing with multiple vibrational modes in a single-step process. These two methods are introduced in what follows.

Single-Mode Tuning Method

The concept of single-mode tuning method depends on the fact that if the frequency of a damper is tuned close to the natural frequency of a target mode with the dashpot removed, the damping coefficient can easily be optimized to eliminate the attendant vibrations most rapidly.[1,5,7] This method can be applied for the design of three inertial dampers onboard a three-cable suspension system for damping augmentation. However, it is restrictive in that each damper is tuned separately with one pendulum mode targeted, so that three sets of parameters, namely m_i, k_i, and c_i, need to be determined independently. The procedure of the single-mode tuning method is outlined as follows: 1) select a proper magnitude for the mass m_i of an inertial damper; 2) select a pendulum mode targeted for such an inertial damper, and determine the value of k_i needed to

Table 1 Model parameters of three-cable suspension system

Geometric parameters	
$l_a = 40.0$ in.	$l_b = 68.1$ in.
$l_c = 85.0$ in.	$l_d = 6.6$ in.
$l_g = 9.7$ in.	
$\phi = 5$ deg	$\phi_1 = 30$ deg
Inertial properties	
$m_1 = 0.001$ lbf	$j_1 = 0.6$ lbm in.^{2}
$m_2 = 1.0$ lbf	$j_2 = 0.01$ lbm in.^{2}
$m_3 = 0.001$ lbf	$j_3 = 0.4$ lbm in.^{2}
$m_4 = 0.001$ lbf	$j_4 = 0.4$ lbm in.^{2}
$m_5 = 1.5$ lbf	$m_6 = 1.5$ lbf
$m_7 = 32.5$ lbf	$j_7 = 7.77$ lbm in.^{2}

match the corresponding natural frequency, viz. $k_i/m_i = \mathcal{F}_i^2$; and 3) determine the value of c_i that optimizes the damping value of the target mode.

The natural frequency required in step 2) can readily be calculated by using Eq. (1) or (2) for the pendulum mode as targeted. To optimize damping, a root locus is drawn for each inertial damper by fixing m_i and k_i and varying c_i, for which an optimal value will coincide with a minimum value of the real part of the eigenvalue of the targeted mode. The eigenvalue of the damped mode can be obtained from Eq. (9) linearized by a suitable linearization code.[11] On applying this single-mode tuning method with design parameters given in Table 1, solution for the damper parameters provides $m_1 = 1.5$ lbf, $k_1 = 1.267$ lbf/ft, $c_1 = 0.217$ lbf/(ft/s), $m_2 = 1.5$ lbf, $k_2 = 1.908$ lbf/ft, $c_2 = 0.174$ lbf/(ft/s), $I_3 = 7.7$ lbf $\cdot$ ft^2, $k_3 = 2.972$ lbf/rad, and $c_3 = 0.326$ lbf/(rad/s). In this way, damping augmentation of the three-cable suspension system is established through the designed dampers.

Multimode Tuning Method

An alternative to the single-mode tuning method for damping augmentation as stated above is a multimode tuning method that can directly cope with the nonlinear equations of motion, and simultaneously offer an optimal solution for the design of three inertial dampers. In the present study, a polytope method[12] is employed to solve an optimization problem by seeking a set of optimum damper parameters for vibration minimization in the three-cable suspension system. The solution procedure is summarized as follows: 1) set up a cost function of discrete results multiplied by time, 2) make the cost function subject to the dynamic equations integrated over a proper range of time, 3) adopt the results in the previous subsection as a feasible starting point for the design parameters, and 4) determine the values of the damper parameters that minimize the cost function. Note that selection of a feasible starting point from the outcome of single-mode tuning method can lead to fast convergence of multimode tuning optimization, since it has been restricted to a region around the optimal solution through the single-mode tuning method.

Based on the minimum-time control concept,[13] the optimization is formulated to minimize the integral of time multiplied by the mean squared value of the vibrational amplitudes:

Minimize

$$F(y) = \int_0^{t_f} t\left[\xi_i^2(t)+\dot{\xi}_i^2(t)\right] \mathrm{d}t = \sum_{i=1}^{n} t_i\left[\xi_i^2(t_i)+\dot{\xi}_i^2(t_i)\right] \Delta t \quad (10)$$

Subject to

$$\phi(y) = \bar{M}\ddot{\xi}_i + \bar{C}\dot{\xi}_i - \bar{U}F = 0 \quad (11)$$

In Eqs. (10) and (11), the optimization program uses the polytope algorithm (IMSL code) to seek the optimal solution of the design variables to minimize a cost function $F(y)$ in a domain with zero lower bounds imposed on the design variables. Throughout the optimization process, the equations of motion given by Eq. (9) are numerically integrated in time to obtain the time-history responses that are discretely sorted at equal intervals for the determination of the cost function defined in Eq. (10), which in turn serves as a constraint imposed upon this optimization problem. However, the constraint for avoiding eigenvalue instability for each new search point is deemed unnecessary, because the cost function in Eq. (10) has been formed quadratically with the purpose of asymptotic stability.[7] Above all, minimization of this cost function is also subject to time optimization, since the time t must be considered as a key index in the cost function, so that the resulting damped vibrations will be decaying as fast as possible.

To implement this optimization program, we specify $t_f = 10.0$ s, $n = 1000$, $\Delta t = 0.01$ s, and the starting point $y^0 = [1.267, 0.217, 1.908, 0.174, 2.972, 0.326]^T$. Two impulsive forces $f_{x2} = 100.0$ lbf and $f_{y2} = 100.0$ lbf are imposed on the end effector of the three-cable suspension system to trigger the pendulum vibrations. At the starting point as chosen, the corresponding starting value of the cost function, $F(y^0)$, equals 20.08, whereas the final cost is 8.04 at the minimum of $F(y)$. The solution to this optimization program has been found to be $y_{\mathrm{opt}} = [1.039, 0.157, 1.18, 0.114, 2.506, 0.202]^T$. With the feasible starting point provided by the single-mode tuning method, it only takes six iterations of optimization for convergence. Moreover, although not reported herein, the optimal solution has been shown to be independent of the different excitations of the system.

Numerical Results

To demonstrate two different tuning methods for damping augmentation of the three-cable suspension system, we have chosen the model parameters for a system equipped with three inertial dampers as summarized in Table 1. The parameters of the inertial dampers for these two different design methods have been obtained in the previous section. An undamped vibrational motion has also been simulated with dampers removed under the same initial condition, for comparison with those using two different sets of damper parameters. In all of them, the impulsive forces $f_{x2} = 100.0$ lbf and $f_{y2} = 100.0$ lbf are initially imposed at the centroid of the end effector to excite the whole system to oscillation, thereby obtaining the transient responses along with a total of three simulations to verify the superiority of the proposed multimode tuning method over others.

Figures 5a–5c are the 10-s numerical results of one undamped and two damped vibration simulations subjected to the initial impulses. The suspended article under the three-cable suspension system

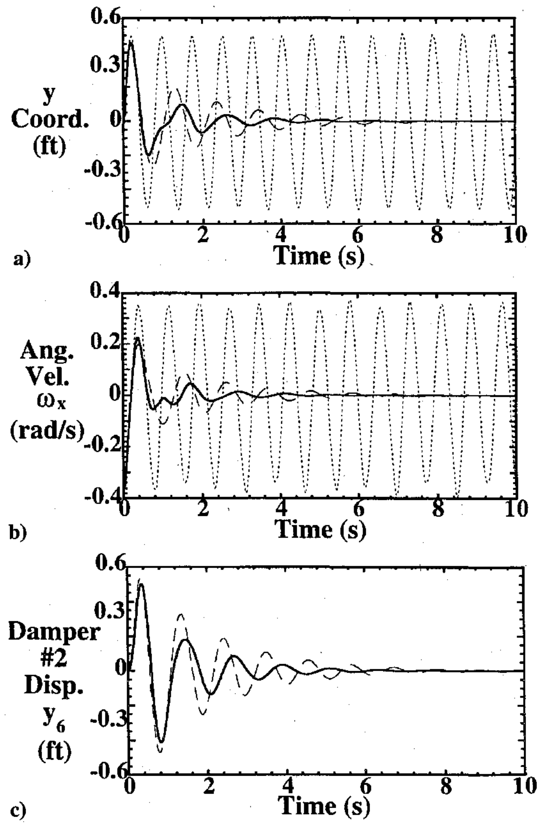


Fig. 5 Simulation results on vibration suppression for a three-cable suspension system.

swings like a single pendulum in space with three pendulum modes superimposed. The results of the multimode tuning method are indicated by a solid line, those of the single-mode tuning method by a dashed line, and those of the undamped case by a dotted line. Figure 5a shows three y components of displacements associated with the centroid of the end effector. As can be seen from the dotted line, the y component fluctuates with the natural frequency of the pitching mode (1.11 Hz), which yields an undamped vibrational motion in the absence of inertial dampers. However, such a vibrational motion is suppressed within a few seconds when three inertial dampers are used. As a result, it can be seen that the solid line is damped out faster than the dashed line, implying that the multimode tuning method is indeed superior to its counterpart for damping augmentation.

Figure 5b, on the other hand, shows the angular velocity of the end effector about the x axis. The phenomenon for this case is consistent with that deduced from the displacement in Fig. 5a. Moreover, the responses of the inertial damper 2 can be observed in Fig. 5c. The impulsive results in Figs. 5a–5c thus are useful in verifying the superiority of the multimode tuning method over the single-mode tuning method. It also shows that the results indeed correspond to the anticipated vibrational characteristics of the three-cable suspension system in three-dimensional space.

Conclusions

The problem of damping augmentation for a three-cable suspension system has been proposed and analyzed through the use of triaxial dampers. Three pendulum modes of the system have been identified, and their natural frequencies derived in closed form. The triaxial dampers have been designed by two methods: 1) tuning each one for a particular mode as targeted and 2) simultaneously tuning the three in a single-step process. The latter uses a multimode tuning approach for the determination of damper parameters, wherein an optimization design is formulated as a minimum-time control problem to achieve the optimality of the damper parameters. The outcome of the first method is used as a feasible starting point for the second method, so that the convergence of the optimization is vastly improved. The inertial dampers as designed in this simultaneous manner have been shown to be applicable and suitable for the damping augmentation of a three-cable suspension system in ground testing of space structures. Comparison of simulation results from the two proposed methods indicates that the triaxial dampers in conjunction with the multimode tuning method have been very effective in reducing vibrations of the three-cable suspension system in minimal time.

Appendix A: Constraint Jacobian Matrices

The constraint Jacobian matrix of a three-cable suspension system can be written as

$$
\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_o \\ \vdots \\ \boldsymbol{B}_c \end{bmatrix} =
$$

$$
\begin{bmatrix}
\boldsymbol{B}_{A1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\boldsymbol{B}_{B1} & -\boldsymbol{B}_{B2} & 0 & 0 & 0 & 0 & 0 \\
0 & \boldsymbol{B}_{C2} & -\boldsymbol{B}_{C3} & 0 & 0 & 0 & 0 \\
0 & \boldsymbol{B}_{F2} & 0 & -\boldsymbol{B}_{F4} & 0 & 0 & 0 \\
0 & \boldsymbol{B}_5 & 0 & 0 & -\boldsymbol{B}_{50} & 0 & 0 \\
0 & \boldsymbol{B}_6 & 0 & 0 & -\boldsymbol{B}_{60} & 0 & 0 \\
0 & \boldsymbol{I} & 0 & 0 & 0 & 0 & -\boldsymbol{B}_{J7} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \boldsymbol{B}_{D3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \boldsymbol{B}_{E4} & 0 & 0 & 0 \\
0 & \boldsymbol{I} & 0 & 0 & -\boldsymbol{I} & 0 & 0 \\
0 & \boldsymbol{I} & 0 & 0 & 0 & -\boldsymbol{I} & 0
\end{bmatrix} \tag{A1}
$$

where

$$
\boldsymbol{B}_{A1} = \left[\boldsymbol{I} \vdots (\tilde{l}_{A1}\boldsymbol{R}_1)^T\right], \qquad \boldsymbol{B}_{B1} = \left[\boldsymbol{I} \vdots (\tilde{l}_{B1}\boldsymbol{R}_1)^T\right]
$$

$$
\boldsymbol{B}_{B2} = \left[\boldsymbol{I} \vdots (\tilde{l}_{B2}\boldsymbol{R}_2)^T\right], \qquad \boldsymbol{B}_{C2} = \left[\boldsymbol{I} \vdots (\tilde{l}_{C2}\boldsymbol{R}_2)^T\right]
$$

$$
\boldsymbol{B}_{C3} = \left[\boldsymbol{I} \vdots (\tilde{l}_{C3}\boldsymbol{R}_3)^T\right], \qquad \boldsymbol{B}_{D3} = \left[\boldsymbol{I} \vdots (\tilde{l}_{D3}\boldsymbol{R}_3)^T\right]
$$

$$
\boldsymbol{B}_{E4} = \left[\boldsymbol{I} \vdots (\tilde{l}_{E4}\boldsymbol{R}_4)^T\right], \qquad \boldsymbol{B}_{F2} = \left[\boldsymbol{I} \vdots (\tilde{l}_{F2}\boldsymbol{R}_2)^T\right]
$$

$$
\boldsymbol{B}_{F4} = \left[\boldsymbol{I} \vdots (\tilde{l}_{F4}\boldsymbol{R}_4)^T\right], \qquad \boldsymbol{B}_{J7} = \left[\boldsymbol{I} \vdots (\tilde{l}_{J7}\boldsymbol{R}_7)^T\right]
$$

$$
\boldsymbol{B}_5 = \left[\tilde{l}_5 \vdots \tilde{d}_5\boldsymbol{R}_2^T\tilde{l}_5\right], \qquad \boldsymbol{B}_6 = \left[\tilde{l}_6 \vdots \tilde{d}_6\boldsymbol{R}_2^T\tilde{l}_6\right]
$$

$$
\boldsymbol{B}_{50} = \left[\tilde{l}_5 \vdots 0\right], \qquad \boldsymbol{B}_{60} = \left[\tilde{l}_6 \vdots 0\right]
$$

In Eq. (A1), $\boldsymbol{I}$ is an identity matrix and $\boldsymbol{R}_i$ is the rotational matrix associated with the ith body. Furthermore, $\tilde{l}_{ij}$ ($i = A, \ldots, F$; $j = 1, \ldots, 7$) are the skew-symmetric matrices formed by the position vectors $\boldsymbol{r}_{ij}$ directed from the c.g. of the ith body to the jth joint on the body-coordinate basis, which can be written as

$$
\tilde{l}_{A1} = \begin{bmatrix} 0 & -l_c/2 & 0 \\ l_c/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{l}_{B1} = \begin{bmatrix} 0 & l_c/2 & 0 \\ -l_c/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\tilde{l}_{B2} = \begin{bmatrix} 0 & -l_g & 0 \\ l_g & 0 & -l_d/\sqrt{3} \\ 0 & l_d/\sqrt{3} & 0 \end{bmatrix}
$$

$$
\tilde{l}_{C2} = \begin{bmatrix} 0 & -l_g & l_d/2 \\ l_g & 0 & l_d/2\sqrt{3} \\ -l_d/2 & -l_d/2\sqrt{3} & 0 \end{bmatrix}
$$

$$
\tilde{l}_{F2} = \begin{bmatrix} 0 & -l_e & -l_d/2 \\ l_e & 0 & l_d/2\sqrt{3} \\ l_d/2 & -l_d/2\sqrt{3} & 0 \end{bmatrix}
$$

$$
\tilde{l}_{C3} = \begin{bmatrix} 0 & l_b/2 & 0 \\ -l_b/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{l}_{D3} = \begin{bmatrix} 0 & -l_b/2 & 0 \\ l_b/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\tilde{l}_{E4} = \begin{bmatrix} 0 & -l_b/2 & 0 \\ l_b/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{l}_{F4} = \begin{bmatrix} 0 & l_b/2 & 0 \\ -l_b/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\tilde{l}_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \qquad \tilde{l}_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}
$$

$$
\tilde{l}_7 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
\tilde{d}_5 = \begin{bmatrix} 0 & -\Delta z_{25} & \Delta y_{25} \\ \Delta z_{25} & 0 & -\Delta x_{25} \\ -\Delta y_{25} & \Delta x_{25} & 0 \end{bmatrix}
$$

$$
\tilde{d}_6 = \begin{bmatrix} 0 & -\Delta z_{26} & \Delta y_{26} \\ \Delta z_{26} & 0 & -\Delta x_{26} \\ -\Delta y_{26} & \Delta x_{26} & 0 \end{bmatrix}, \qquad \tilde{l}_{J7} = \begin{bmatrix} 0 & -l_g & 0 \\ l_g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

The variable transformation matrix $\boldsymbol{A}_{\mathrm{o}}$ is given by

$$\boldsymbol{A}_{\mathrm{o}} =$$

$$\begin{bmatrix} -\boldsymbol{B}_{A1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & -\boldsymbol{B}_{B2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{I} & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & \boldsymbol{B}_{C2}-\boldsymbol{B}_{B2} & -\boldsymbol{B}_{C3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{I} & 0 & 0 & 0 & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & \boldsymbol{B}_{F2}-\boldsymbol{B}_{B2} & 0 & -\boldsymbol{B}_{F4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \boldsymbol{I} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & \boldsymbol{B}_{5}-\boldsymbol{B}_{B2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boldsymbol{I} & 0 & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & -\boldsymbol{B}_{6}-\boldsymbol{B}_{B2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boldsymbol{I} & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & -\boldsymbol{B}_{B2} & 0 & 0 & 0 & 0 & -\boldsymbol{B}_{J7} \\ 0 & 0 & 0 & 0 & 0 & 0 & \boldsymbol{I} \end{bmatrix} \tag{A2}$$

The remaining Jacobian matrix $\boldsymbol{B}_{\mathrm{n}}$ becomes

$$\boldsymbol{B}_{\mathrm{n}} = \boldsymbol{B}_{c}\boldsymbol{A}_{\mathrm{o}} =$$

$$\begin{bmatrix} \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & \boldsymbol{B}_{C2}-\boldsymbol{B}_{B2} & \boldsymbol{B}_{D3}-\boldsymbol{B}_{C3} & 0 & 0 & 0 & 0 \\ \boldsymbol{B}_{B1}-\boldsymbol{B}_{A1} & \boldsymbol{B}_{F2}-\boldsymbol{B}_{B2} & 0 & \boldsymbol{B}_{E4}-\boldsymbol{B}_{F4} & 0 & 0 & 0 \\ 0 & \boldsymbol{I} & 0 & 0 & -\boldsymbol{I} & 0 & 0 \\ 0 & \boldsymbol{I} & 0 & 0 & 0 & -\boldsymbol{I} & 0 \\ 0 & l_7 & 0 & 0 & 0 & -l_7 & 0 \end{bmatrix} \tag{A3}$$

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